SU(3) Einstein-Yang-Mills-Dilaton Sphalerons and Black Holes

Burkhard Kleihaus[1], **Jutta Kunz**[1,2] **and Abha Sood**[1]
[1]Fachbereich Physik, Universität Oldenburg, Postfach 2503
D-26111 Oldenburg, Germany
[2] Instituut voor Theoretische Fysica, Rijksuniversiteit te Utrecht
NL-3508 TA Utrecht, The Netherlands

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Abstract

SU(3) Einstein-Yang-Mills-dilaton theory possesses sequences of static spherically symmetric sphaleron and black hole solutions for the SU(2) and the SO(3) embedding. The solutions depend on the dilaton coupling constant γ, approaching the corresponding Einstein-Yang-Mills solutions for $\gamma \to 0$, and Yang-Mills-dilaton solutions in flat space for $\gamma \to \infty$. The sequences of solutions tend to Einstein-Maxwell-dilaton solutions with different magnetic charges. The solutions satisfy analogous relations between the dilaton field and the metric for general γ. Thermodynamic properties of the black hole solutions are discussed.

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1 Introduction

SU(2) Yang-Mills theory in $3+1$ dimensions does not possess static, finite energy solutions. In contrast, SU(2) Einstein-Yang-Mills theory possesses a sequence of static, spherically symmetric sphaleron solutions [1, 2, 3], and so does SU(2) Yang-Mills-dilaton theory [4, 5] as well as SU(2) Yang-Mills-Higgs theory, where the electroweak sphaleron [6, 7, 8] is accompanied by such a sequence for large Higgs boson masses [9, 10].

Motivated by higher-dimensional unified theories, such as Kaluza-Klein theory or superstring theory, where a scalar dilaton field arises naturally, recently SU(2) Einstein-Yang-Mills-dilaton theory has been studied and shown to also possess a sequence of static spherically symmetric regular particle-like solutions which are unstable [11, 12, 13, 14, 15].

Like SU(2) Einstein-Yang-Mills theory [16, 17, 18], SU(2) Einstein-Yang-Mills-dilaton theory additionally possesses a sequence of static spherically symmetric black hole solutions with non-trivial matter fields [11, 12, 13, 15]. The sequences of black hole solutions are unstable, too [19, 20, 21, 12].

Here we consider SU(3) Einstein-Yang-Mills-dilaton theory. The equations of motion obtained for static spherically symmetric ansätze for the metric and for the matter fields (with magnetic gauge field) yield relations between the dilaton field and the metric functions. We construct sphaleron and black hole solutions, employing both the SU(2) and the SO(3) embedding. The solutions are similar to the corresponding solutions of SU(3) Einstein-Yang-Mills theory [22, 23], and approach these for small dilaton coupling constant γ.

Labeling the solutions by the number of nodes of the gauge field functions, n, we construct the sphaleron and black hole solutions with four or less nodes. We relate the properties of the regular and black hole solutions in the limit $n \to \infty$ to those of black hole solutions of Einstein-Maxwell-dilaton theory [24, 25].

2 SU(3) Einstein-Yang-Mills-Dilaton Equations of Motion

We consider the SU(3) Einstein-Yang-Mills-dilaton action

$$S = S_G + S_M = \int L_G \sqrt{-g} d^4x + \int L_M \sqrt{-g} d^4x \tag{1}$$

with

$$L_G = \frac{1}{16\pi G} R \ , \tag{2}$$

and the matter Lagrangian

$$L_M = -\frac{1}{2}\partial_\mu \Phi \partial^\mu \Phi - e^{2\kappa\Phi}\frac{1}{2}\mathrm{Tr}(F_{\mu\nu}F^{\mu\nu}) \ , \tag{3}$$

where

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu] \ , \tag{4}$$

$$A_\mu = \frac{1}{2}\lambda^a A_\mu^a \ , \tag{5}$$

and g, κ are the gauge and dilaton coupling constants, respectively. Variation of the action eq. (1) with respect to the metric $g_{\mu\nu}$ leads to the Einstein equations, and variation with respect to the gauge field A_μ and the dilaton field Φ to the matter field equations.

To construct static spherically symmetric regular and black hole solutions we employ Schwarzschild-like coordinates and adopt the spherically symmetric metric

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = -A^2 N dt^2 + N^{-1}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2) \ , \tag{6}$$

with

$$N = 1 - \frac{2m}{r} \ . \tag{7}$$

Generalized spherical symmetry for the gauge field is realized by embedding the SU(2) or the SO(3) generators in SU(3). In the SU(2)-embedding the ansatz for the gauge field with vanishing time component is [1],

$$\begin{aligned} A_0 &= 0 \ , \\ A_i &= \frac{1-w(r)}{2rg}(\vec{e}_r \times \vec{\tau})_i \ , \end{aligned} \tag{8}$$

with the SU(2) Pauli matrices $\vec{\tau} = (\tau_1, \tau_2, \tau_3)$. In the SO(3)-embedding the corresponding ansatz for the gauge field with vanishing time component is

$$\begin{aligned} A_0 &= 0 \ , \\ A_i &= \frac{2-K(r)}{2rg}(\vec{e}_r \times \vec{\Lambda})_i + \frac{H(r)}{2rg}\Big[(\vec{e}_r \times \vec{\Lambda})_i, \vec{e}_r \cdot \vec{\Lambda})\Big]_+ \ , \end{aligned} \tag{9}$$

where $[\ ,\ ]_+$ denotes the anticommutator, and $\vec{\Lambda} = (\lambda_7, -\lambda_5, \lambda_2)$. For the dilaton field we take $\Phi = \Phi(r)$.

The SU(2)-embedding, eq. (8), leads to the previously studied SU(2) Einstein-Yang-Mills-dilaton equations [11, 12, 13, 14, 15]. To obtain the SU(3) Einstein-Yang-Mills-dilaton equations for the SO(3)-embedding, eq. (9), we employ the tt and rr components of the Einstein equations, yielding for the metric functions

$$\mu' = \frac{1}{2}Nx^2\phi'^2 + e^{2\gamma\phi}\left[N(K'^2 + H'^2) + \frac{1}{8x^2}\left(\left(K^2 + H^2 - 4\right)^2 + 12K^2H^2\right)\right] \ , \tag{10}$$

$$A' = \frac{2}{x}\left[\frac{1}{2}x^2\phi'^2 + e^{2\gamma\phi}(K'^2 + H'^2)\right]A \ . \tag{11}$$

Here we have introduced the dimensionless coordinate $x = (g/\sqrt{4\pi G})r$ (the prime indicates the derivative with respect to x), the dimensionless mass function

$$\mu = \frac{g}{\sqrt{4\pi G}} m = \frac{g m_{\rm Pl}}{\sqrt{4\pi}} m \ , \tag{12}$$

the dimensionless dilaton field $\phi = \sqrt{4\pi G}\Phi$, and the dimensionless coupling constant $\gamma = \kappa/\sqrt{4\pi G}$. The choice $\gamma = 1$ corresponds to string theory, while $4+n$ dimensional Kaluza-Klein theory has $\gamma^2 = (2+n)/n$ [24].

For the matter field functions we obtain the equations

$$(e^{2\gamma\phi} ANK')' = e^{2\gamma\phi} \frac{1}{4x^2} AK \left(K^2 + 7H^2 - 4 \right) \ , \tag{13}$$

$$(e^{2\gamma\phi} ANH')' = e^{2\gamma\phi} \frac{1}{4x^2} AH \left(H^2 + 7K^2 - 4 \right) \ , \tag{14}$$

$$(ANx^2\phi')' = 2\gamma A e^{2\gamma\phi} \left[N(K'^2 + H'^2) + \frac{1}{8x^2} \left(\left(K^2 + H^2 - 4 \right)^2 + 12K^2H^2 \right) \right] \ . \tag{15}$$

With help of eq. (11) the metric function A can be eliminated from the matter field equations. Note, that the equations are symmetric with respect to an interchange of the functions $K(x)$ and $H(x)$, and to the transformations $K(x) \rightarrow -K(x)$, and $H(x) \rightarrow -H(x)$, yielding degenerate solutions.

Let us now derive relations between the dilaton field and the metric functions. We first note, that the terms containing gauge field functions in eq. (10) for the metric function μ and in eq. (15) for the dilaton function ϕ agree, allowing us to replace the gauge field term in the dilaton field equation. We further note, that the expression for the curvature scalar, obtained from the contracted Einstein equations, does not involve the gauge field,

$$\frac{1}{2} A r^2 R = ANx^2 \phi'^2 \ . \tag{16}$$

With help of this relation we then find for the dilaton field the equation

$$(ANx^2\phi')' = \frac{1}{2}\gamma \left(x^2 (2A'N + AN') \right)' \ , \tag{17}$$

or, after integration,

$$\phi' = \frac{1}{2}\gamma \left(\ln(A^2N) \right)' + \frac{C}{ANx^2} \ , \tag{18}$$

where C is an integration constant. Note, that these relations hold in general for static, spherically symmetric solutions with magnetic gauge fields (and analogous relations for electric gauge fields). We return to the relations (17) and (18) in the next sections after fixing the boundary conditions.

As in Einstein-Yang-Mills theory, comparison of the equations of the SO(3) embedding and those of the SU(2) embedding [11, 12, 13, 14, 15] shows, that to each SU(2) solution there corresponds a "scaled SU(2)" solution of the SO(3) system with precisely double the mass of its SU(2) counterpart. (With $x = 2\tilde{x}$, $\mu(x) = 2\tilde{\mu}(\tilde{x})$, $K(x) = 2w(\tilde{x})$, $H(x) = 0$ and $\phi(x) = \tilde{\phi}(\tilde{x})$ the functions $\tilde{\mu}$, w and $\tilde{\phi}$ satisfy the SU(2) equations with coordinate $\tilde{x}$.)

3 Regular Solutions

Let us first consider the regular solutions of SU(3) Einstein-Yang-Mills-dilaton theory. Requiring asymptotically flat solutions implies that the metric functions A and μ both approach a constant at infinity. We here adopt

$$A(\infty) = 1 \ , \tag{19}$$

thus fixing the time coordinate. For magnetically neutral solutions the gauge field functions approach a vacuum configuration

$$K(\infty) = \pm 2 \ , \quad H(\infty) = 0 \ , \tag{20}$$

$$K(\infty) = 0 \ , \quad H(\infty) = \pm 2 \ . \tag{21}$$

For the dilaton field we choose [11, 12, 14, 15]

$$\phi(\infty) = 0 \ . \tag{22}$$

At the origin regularity of the solutions requires

$$\mu(0) = 0 \ , \tag{23}$$

the gauge field functions must satisfy

$$K(0) = \pm 2 \ , \quad H(0) = 0 \ , \tag{24}$$

$$K(0) = 0 \ , \quad H(0) = \pm 2 \ , \tag{25}$$

while the dilaton field satisfies

$$\phi'(0) = 0 \ . \tag{26}$$

As in Einstein-Yang-Mills theory [23] it is sufficient to consider solutions with $K(0) = 2$ and $H(0) = 0$.

We now return to the relations between the metric and the dilaton field, eqs. (17) and (18). Defining the dilaton charge D via

$$\phi(x) \stackrel{x\to\infty}{\longrightarrow} -\frac{D}{x} \ , \tag{27}$$

eq. (17) connects the dilaton charge and the mass of the solution. By integrating eq. (17) from zero to infinity we obtain the relation

$$D = \gamma\mu(\infty) \ . \tag{28}$$

Consequently the integration constant in eq. (18) vanishes, $C = D - \gamma\mu(\infty)$. Integrating eq. (18) from x to infinity then gives the relation

$$\phi(x) = \frac{1}{2}\gamma\ln(A^2 N) = \frac{1}{2}\gamma\ln(-g_{tt}) \ . \tag{29}$$

These results (28) and (29) generalize the relations obtained previously for SU(2) and $\gamma = 1$ [11, 14, 15, 26][1]. They are valid for general static, spherically symmetric, magnetic gauge fields.

In the following we present numerical results for the regular solutions of the SO(3) embedding. In Figs. 1-4 we show the lowest SO(3) solution for $\gamma = 0$, 1, 2 and 4. The excited solutions will be shown elsewhere [27]. In the limit of vanishing coupling constant γ the solutions approach smoothly the corresponding Einstein-Yang-Mills solutions [23]. In the limit $\gamma \to \infty$, on the other hand, the solutions approach those of SU(3) Yang-Mills-dilaton theory in flat space.

In Fig. 5 we show the value of the dilaton field at the origin as a function of the coupling constant γ, for the lowest SO(3) and SU(2) solutions. While in the limits $\gamma \to 0$ and $\gamma \to \infty$ the value of the dilaton field at the origin approaches zero, a minimum occurs at $\gamma = 0.92$ for the SO(3) solutions, close to the minimum of the SU(2) solutions at $\gamma = 0.91$. Remember, that the dilaton function is related to the metric via eq. (29).

To discuss the sequences of solutions, we adopt the classification of the solutions with respect to the number of nodes (n_1, n_2) of the functions (u_1, u_2) [22, 23],

$$u_1(x) = \frac{K(x) + H(x)}{2} \ , \quad u_2(x) = \frac{K(x) - H(x)}{2} \ , \tag{30}$$

and the total number of nodes n. ($n = n_1 + n_2$ for SO(3) solutions.) The lowest SO(3) solution has $n = 1$ and node structure $(0, 1)$, while the lowest scaled SU(2) solution has $n = 2$ and node structure $(1, 1)$.

As observed in [12, 14], in the limit $n \to \infty$ the sequence of regular SU(2) solutions for a given γ tends to the "extremal" Einstein-Maxwell-dilaton black hole [24, 25] with the same γ and with magnetic charge $P = 1$. "Extremal" Einstein-Maxwell-dilaton black hole solutions satisfy the relation between mass and magnetic charge, $\mu = P/\sqrt{1+\gamma^2}$. The sequences of scaled SU(2) solutions, having twice the mass and dilaton charge of the corresponding SU(2) solutions, therefore approach the "extremal"

[1] Ref. [14] contains a scaled version of relation (28) for general γ.

Einstein-Maxwell-dilaton black holes with twice the magnetic charge, $P = 2$. Consequently, the limiting mass of this sequence of solutions with node structure (n, n) is given by $\mu = 2/\sqrt{1+\gamma^2}$.

In Table 1 the dimensionless mass $\mu(\infty)$ of the lowest SO(3) solutions with four or less nodes is shown for the values of the dilaton coupling constant $\gamma = 0$, 1, 3, and 10. Also shown is the limiting case $\gamma = \infty$. This flat space limit is obtained by scaling the coordinate, the mass and the dilaton field according to $x = \bar{x}\gamma$, $\mu = \bar{\mu}/\gamma$, and $\phi = \bar{\phi}/\gamma$. The ADM mass of the solutions is given by $m_{\rm ADM} = \mu(\infty)\sqrt{4\pi}m_{\rm Pl}/g$.

Let us now inspect the sequence of solutions with node structure $(0, n)$, including the lowest SO(3) solution. The masses $\mu(\infty)$ of these solutions, shown in Table 1 for $n = 1 - 4$, are well approximated by the formula

$$\mu(\infty, \gamma, n) = \frac{\sqrt{3} - \frac{\pi}{2}e^{-\frac{4}{3}n}}{\sqrt{1+\gamma^2}} \ . \tag{31}$$

They approach for large n the (extrapolated) limiting value $\sqrt{3/(1+\gamma^2)}$, shown for comparison. This indicates, that in the limit $n \to \infty$ the regular SO(3) solutions with node structure $(0, n)$ tend to the "extremal" Einstein-Maxwell-dilaton black holes [24, 25] with magnetic charge $P = \sqrt{3}$ [27].

The order of the solutions with respect to their mass does not depend on the dilaton coupling constant. For a given total number of nodes, the $(0, n)$ solutions are the lowest solutions [23], as seen in Table 1, and their limiting mass is lower than the limiting mass of the (n, n) solutions. Further details will be given elsewhere [27]. The limiting behaviour of solutions with general node structure (n_1, n_2) is still open, since excited solutions are difficult to obtain.

The regular SU(2) Einstein-Yang-Mills-dilaton solutions are unstable [12, 14], so are the regular SO(3) Einstein-Yang-Mills solutions, obtained in the limit $\gamma \to 0$ [23, 28]. By continuity, we conclude, that the regular SO(3) Einstein-Yang-Mills-dilaton solutions are unstable, too.

4 Black Hole Solutions

We now turn to the black hole solutions of SU(3) Einstein-Yang-Mills-dilaton theory. Imposing again the condition of asymptotic flatness, the black hole solutions considered here satisfy the same boundary conditions at infinity as the regular solutions. The existence of a regular event horizon at $x_{\rm H}$ requires

$$\mu(x_{\rm H}) = \frac{x_{\rm H}}{2} \ , \tag{32}$$

and $A(x_{\rm H}) < \infty$, and the matter functions at the horizon $x_{\rm H}$ must satisfy

$$N'K' = \frac{1}{4x^2} K \left(K^2 + 7H^2 - 4 \right) \ , \tag{33}$$

$$N'H' = \frac{1}{4x^2} H \left(H^2 + 7K^2 - 4 \right) \ , \tag{34}$$

$$N'\phi' = \gamma e^{2\gamma\phi} \left[\frac{1}{4x^4} \left(\left(K^2 + H^2 - 4 \right)^2 + 12K^2H^2 \right) \right] \ . \tag{35}$$

Note that a restricted subset of Einstein-Yang-Mills-dilaton black hole solutions with $K \equiv H$ (and thus different boundary conditions), corresponding to magnetically charged black holes, was obtained previously [26]. Unlike the magnetically neutral black hole solutions considered here, however, these solutions have no regular limit [29].

Let us introduce the dimensionless Hawking temperature T for the metric (6) [11, 13]

$$T = T_{\rm S} A (1 - 2\mu')|_{x_{\rm H}} = T_{\rm S} x_{\rm H} A N' |_{x_{\rm H}} \ , \tag{36}$$

where $T_{\rm S} = (4\pi x_{\rm H})^{-1}$ is the Hawking temperature of the Schwarzschild black hole. Let us now consider the relations (17) and (18) for black holes. Integrating eq. (17) from the horizon to infinity yields the relation

$$D = \gamma \left(\mu(\infty) - \frac{1}{2} x_{\rm H}^2 A N' |_{x_{\rm H}} \right) = \gamma \mu(\infty) \left(1 - \frac{\mu_{\rm S}}{\mu(\infty)} \frac{T}{T_{\rm S}} \right) \ , \tag{37}$$

where $\mu_{\rm S} = x_{\rm H}/2$ is the mass of the Schwarzschild black hole. An equivalent expression was obtained for SU(2) black holes for $\gamma = 1$ [15]. Again relation (37) holds for general static, spherically symmetric, magnetic gauge fields, i. e. beside the magnetically neutral SU(2) and SO(3) black holes considered here, it also holds for the magnetically charged Einstein-Maxwell-dilaton black holes [24, 25] and SU(3) Einstein-Yang-Mills dilaton black holes [26]. Unlike the regular case, the integration constant $C = D - \gamma\mu(\infty)$ here in general does not vanish[2], and integration of eq. (18) does not yield a simple expression.

The SO(3) Einstein-Yang-Mills-dilaton black hole solutions are similar to the Einstein-Yang-Mills black hole solutions [23] and approach these smoothly in the limit $\gamma \to 0$. As examples we show the radial functions for the lowest SO(3) black hole solutions with horizon $x_{\rm H} = 3$ and $\gamma = 0$, 1, 2 and 4 in Figs. (1)-(4). In Fig. (5) we show the dilaton field at the horizon. With increasing horizon the minimum of $\phi(x_{\rm H})$ shifts to larger values of γ. Further details of these solutions and the excited SO(3) Einstein-Yang-Mills-dilaton black holes will be shown elsewhere [27].

The families of SO(3) Einstein-Yang-Mills-dilaton black hole solutions change continuously as a function of the horizon $x_{\rm H}$. For vanishing horizon, $x_{\rm H} \to 0$, the black hole

[2] An exception are the "extremal" magnetically charged Einstein-Maxwell-dilaton black holes [24, 25].

solutions approach the regular solutions. With increasing horizon $x_{\rm H}$ the black hole solutions keep their identity in terms of the boundary conditions and the node structure. Only the excited solutions with node structure (n,n), having a slightly higher mass than the corresponding scaled SU(2) solutions, merge at some critical values of the horizon into the scaled SU(2) solutions [23]. These critical values now depend on the dilaton coupling constant. The critical value of the $(1,1)$ solution, for instance, first decreases for small values of γ, reaches a minimum at about $\gamma=1$, and then increases. Note that the order of the solutions (Table 2) changes from the order of the regular solutions as the horizon increases.

In Table 2 we present properties of black hole solutions with horizon $x_{\rm H}=1$, emerging from the lowest regular solutions. The table gives the mass $\mu(\infty)$, the dilaton charge D, and the ratio of the Hawking temperature to the Schwarzschild Hawking temperature, for the dilaton coupling constants $\gamma=1$ and 10. As for the regular solutions, we observe two different sets of limiting values for the sequences of black hole solutions with node structure $(0,n)$ and (n,n).

Though not observed previously, the generalization of the limiting behaviour from the regular solutions to the black hole solutions is straightforward. The sequences of black hole solutions tend to Einstein-Maxwell-dilaton black hole solutions with the same γ and the same horizon, and with magnetic charge $P=1$ for the SU(2) solutions, and magnetic charges $P=\sqrt{3}$ and $P=2$ for the $(0,n)$ and (n,n) SO(3) solutions, respectively. The limiting values are also shown in Table 2[3]. For $\gamma=1$ simple formulae hold for mass, dilaton charge and Hawking temperature of the limiting black hole solutions [24, 25]. Denoting $r_+=\sqrt{x_{\rm H}^2+2P^2}$ one obtains $\mu(\infty)=r_+/2$, $D=P^2/r_+$ and $T/T_{\rm S}=x_{\rm H}/r_+$.

Let us now turn to the thermodynamic properties of the black hole solutions. In Fig. 6 we show the dimensionless inverse Hawking temperature of the lowest black hole solutions for SU(2) and SO(3) as a function of their mass, for the dilaton coupling constants $\gamma=0$, 0.5 and 1. Also shown are the corresponding Schwarzschild and Reissner-Nordstrøm inverse Hawking temperatures. As in the SU(2) case [13][4], we observe, depending on the dilaton coupling constant, phase transitions. For vanishing γ, there are two critical values of the mass $\mu_1=1.561$ and $\mu_2=1.588$ for SO(3) and $\mu_1=0.905$ and $\mu_2=1.061$ for SU(2) [13]. The critical behaviour disappears beyond $\gamma=0.0293$ for SO(3) and $\gamma=0.38$ for SU(2). The thermodynamical properties of the excited solutions tend with increasing n to those of the corresponding Einstein-Maxwell-dilaton black holes with magnetic charges $P=1$ for SU(2) and $P=\sqrt{3}$ and $P=2$ for the $(0,n)$ and (n,n) SO(3) sequences, respectively, while for $\gamma=0$ they tend to those of the corresponding Reissner-Nordstrøm black holes [27]. Further details will

[3] The properties of the limiting charged SU(3) solutions [26] are obtained with $P=1$, not $P=\sqrt{3}/2$.

[4] The coupling of ref. [13] is related to ours via $\kappa=\sqrt{2}\gamma$.

be given elsewhere [27].

The SU(2) Einstein-Yang-Mills-dilaton black holes are unstable [12], so are the SO(3) Einstein-Yang-Mills black holes obtained in the limit $\gamma \to 0$ [23, 28]. Again we conclude by continuity, that the SO(3) Einstein-Yang-Mills-dilaton black hole solutions are also unstable.

5 Conclusion

Classical solutions of SU(3) Einstein-Yang-Mills-dilaton theory are very similar to those of SU(3) Einstein-Yang-Mills theory [23]. There are sequences of regular and black hole solutions based on static, spherically symmetric, magnetic ansätze for the SO(3) embedding as well as the previously studied SU(2) embedding [11, 12, 13, 14, 15].

For the regular solutions two relations between metric and dilaton field hold, $D = \gamma\mu(\infty)$ and $\phi(x) = \gamma \ln(\sqrt{-g_{tt}})$, whereas the black hole solutions satisfy the relation between mass and dilaton charge, $D = \gamma(\mu(\infty) - 2\pi x_{\rm H}^2 T)$, where T is the Hawking temperature. These relations holds in general for static, spherically symmetric solutions with magnetic gauge fields.

The SO(3) solutions can be labeled according to the node structure of the gauge field functions by the integers (n_1, n_2) and the total number of nodes n. The lowest solutions have node structure $(0,1)$ and $(0,2)$, being of type $(0,n)$. The next two solutions both have node structure $(1,1)$, corresponding to the lowest scaled SU(2) solution and an excitation.

For large n the solutions approach limiting solutions. In the limit $n \to \infty$, mass and dilaton charge of the regular excited SO(3) solutions with node structure $(0,n)$ and (n,n) tend to those of the "extremal" Einstein-Maxwell-dilaton solutions with magnetic charge $P = \sqrt{3}$ and $P = 2$, respectively, while mass and dilaton charge of the regular excited SU(2) solutions tend to those of the "extremal" Einstein-Maxwell-dilaton solutions with magnetic charge $P = 1$.

For the black hole solutions the limiting behaviour is analogous. In the limit $n \to \infty$ mass, dilaton charge and Hawking temperature of a sequence of excited black hole solutions with a given horizon tend to those of the corresponding Einstein-Maxwell-dilaton solution with the same horizon. (The limiting behaviour of the metric and matter functions will be discussed elsewhere [27].)

The lowest SO(3) Einstein-Yang-Mills black holes have similar thermodynamic properties as their SU(2) counterparts [13]. For small γ, there are two critical masses associated with two phase transitions, where the specific heat changes sign. The thermodynamic properties of the excited black hole solutions tend to those of the corresponding Einstein-Maxwell-dilaton solutions with magnetic charges $P = 1$, $P = \sqrt{3}$ and $P = 2$ for the SU(2) and SO(3) $(0,n)$ and (n,n) solutions, respectively.

We finally remark that the SO(3) Einstein-Yang-Mills-dilaton black hole solutions constructed here constitute counterexamples to the “no-hair conjecture”, since SU(3) Einstein-Yang-Mills-dilaton theory also contains Schwarzschild black holes. However, only the Schwarzschild black holes are stable.

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#	nodes		$\gamma =$	$\mu(\infty)$				$\bar{\mu}(\infty)$
	u_1	u_2		0.	1.	3.	10.	∞
1	0	1		1.30778	0.90853	0.40083	0.12575	1.26319
2	0	2		1.62261	1.14153	0.50845	0.15984	1.60603
3^*	1	1		1.65729	1.15397	0.50989	0.16001	1.60753
4	1	1		1.69538	1.18798	0.52760	0.16576	1.66536
5	0	3		1.70474	1.20383	0.53779	0.16918	1.69998
6	0	4		1.72531	1.21958	0.54526	0.17156	1.72393
l_1	0	∞		1.73205	1.22474	0.54772	0.17235	1.73205
7	1	2		1.87547	1.32130	0.58916	0.18526	1.86143
8	1	3		1.92840	1.36196	0.60850	0.19143	1.92350
9^*	2	2		1.94269	1.36967	0.61107	0.19217	1.93108
10	2	2		1.94707	1.37417	0.61363	0.19302	1.93963
l_2	∞	∞		2.00000	1.41421	0.63246	0.19901	2.00000

Table 1
The dimensionless mass $\mu(\infty)$ of the lowest regular SO(3) solutions with four or less nodes for dilaton coupling constants $\gamma = 0, 1, 3, 10$, and the scaled mass $\bar{\mu}(\infty)$, obtained in the flat space limit $\gamma \to \infty$. The lines l_1 and l_2 give the limiting values $\sqrt{3/(1+\gamma^2)}$ and $\sqrt{4/(1+\gamma^2)}$ of the $(0,n)$ and (n,n) sequences, respectively. The $*$ indicates the scaled SU(2) solutions. The numbers # refer to the order of these solutions with respect to their mass.

#	$\gamma =$	$\mu(\infty)$		$\gamma =$	D		$\gamma =$	T/T_S	
		1.	10.		1.	10.		1.	10.
1		1.16497	.61858		.87027	1.25371		.58942	.98641
2		1.30296	.64749		1.08770	1.58722		.43054	.97754
3*		1.38135	.65196		1.12231	1.59670		.51809	.98458
4		1.38437	.65546		1.13889	1.64944		.49097	.98104
5		1.32140	.65284		1.12979	1.66560		.38322	.97255
6		1.32279	.65335		1.13364	1.67533		.37830	.97163
l_1		1.32288	.65339		1.13389	1.67603		.37796	.97157
7		1.46227	.67144		1.26493	1.83794		.39467	.97529
8		1.47209	.67440		1.28839	1.88424		.36740	.97195
9*		1.49382	.67739		1.31695	1.90744		.35375	.97329
l_2		1.50000	.67936		1.33333	1.94262		.33333	.97020

Table 2
The dimensionless mass $\mu(\infty)$, dilaton charge D and ratio of the Hawking temperature to the Schwarzschild Hawking temperature $T/T_{\rm S}$ of the lowest black hole solutions with four or less nodes for dilaton coupling constants $\gamma = 1$ and 10 and horizon $x_{\rm H} = 1$. The lines l_1 and l_2 give the limiting values of the $(0, n)$ and (n, n) sequences, respectively. The $*$ indicates the scaled SU(2) solutions. The numbers # correspond to those of the regular solutions (obtained for $x_{\rm H} \to 0$). Note that the black hole solution #10 with node structure $(2, 2)$ does not exist for $0.4 < \gamma < 10.5$ and horizon $x_{\rm H} = 1$.

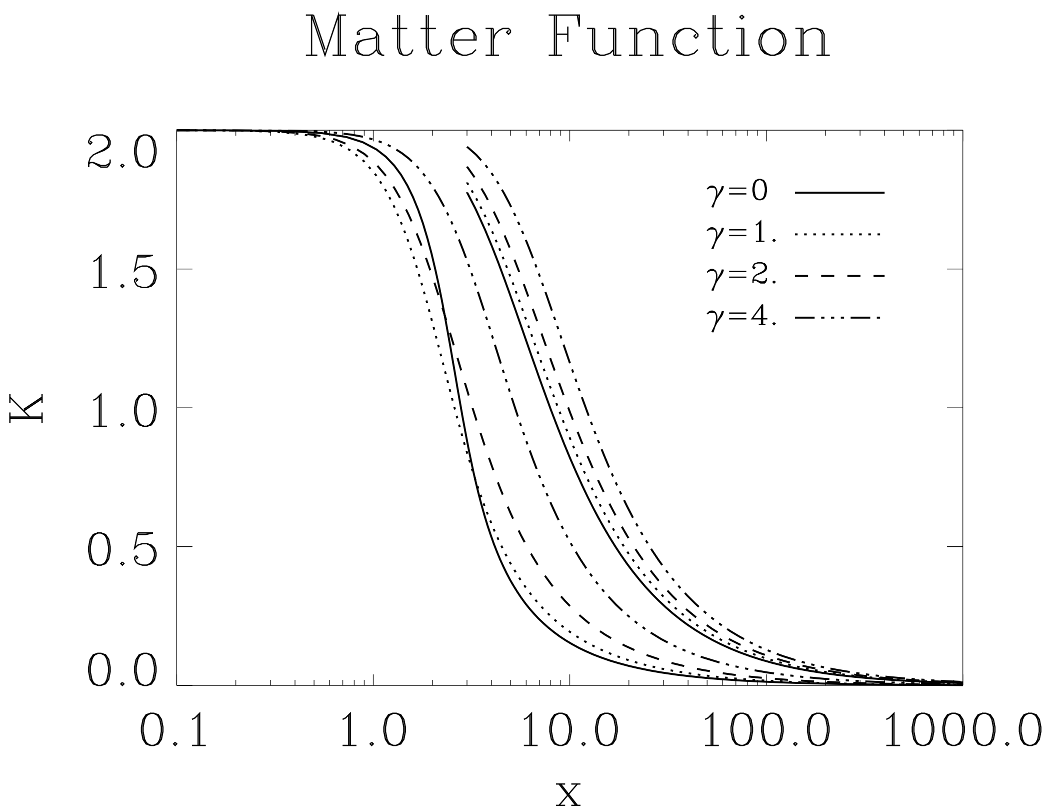

Figure 1: The function $K(x)$ is shown for the regular solution and for the black hole solution with horizon $x_{\rm H} = 3$ for the dilaton coupling constants $\gamma = 0$ (solid), $\gamma = 1$ (dotted), $\gamma = 2$ (dashed) and $\gamma = 4$ (tripledot-dashed) as a function of x.

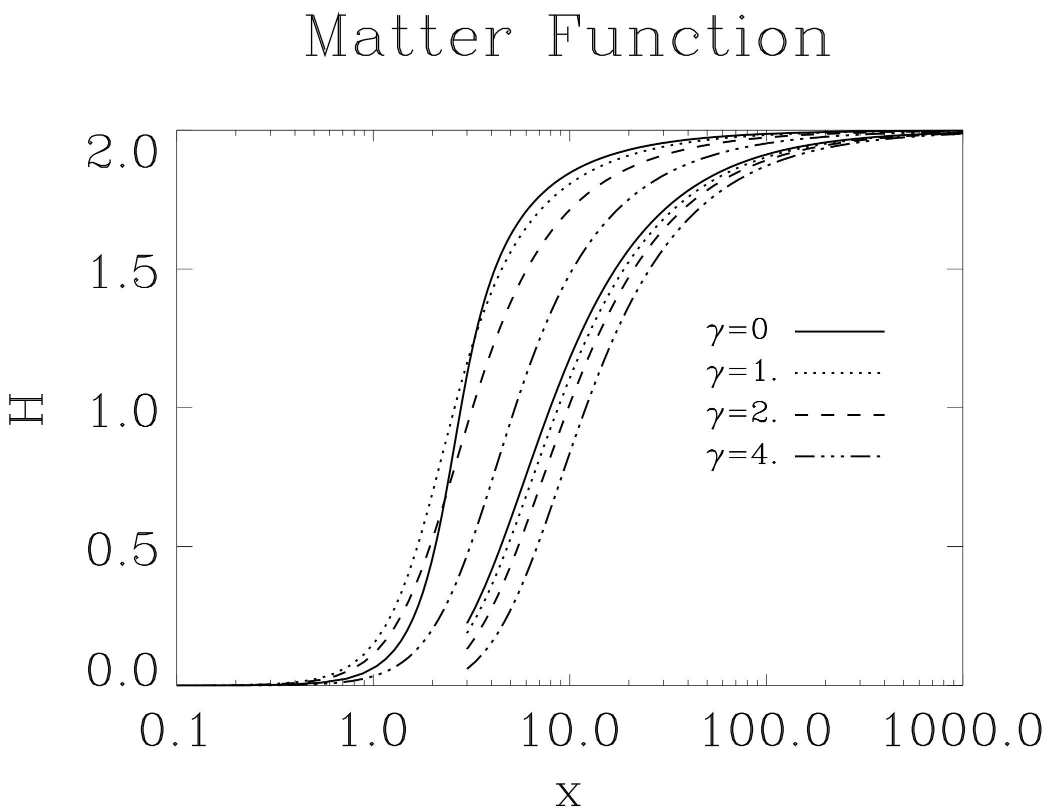

Figure 2: The function $H(x)$ is shown for the regular solution and for the black hole solution with horizon $x_{\rm H} = 3$ for the dilaton coupling constants $\gamma = 0$ (solid), $\gamma = 1$ (dotted), $\gamma = 2$ (dashed) and $\gamma = 4$ (tripledot-dashed) as a function of x.

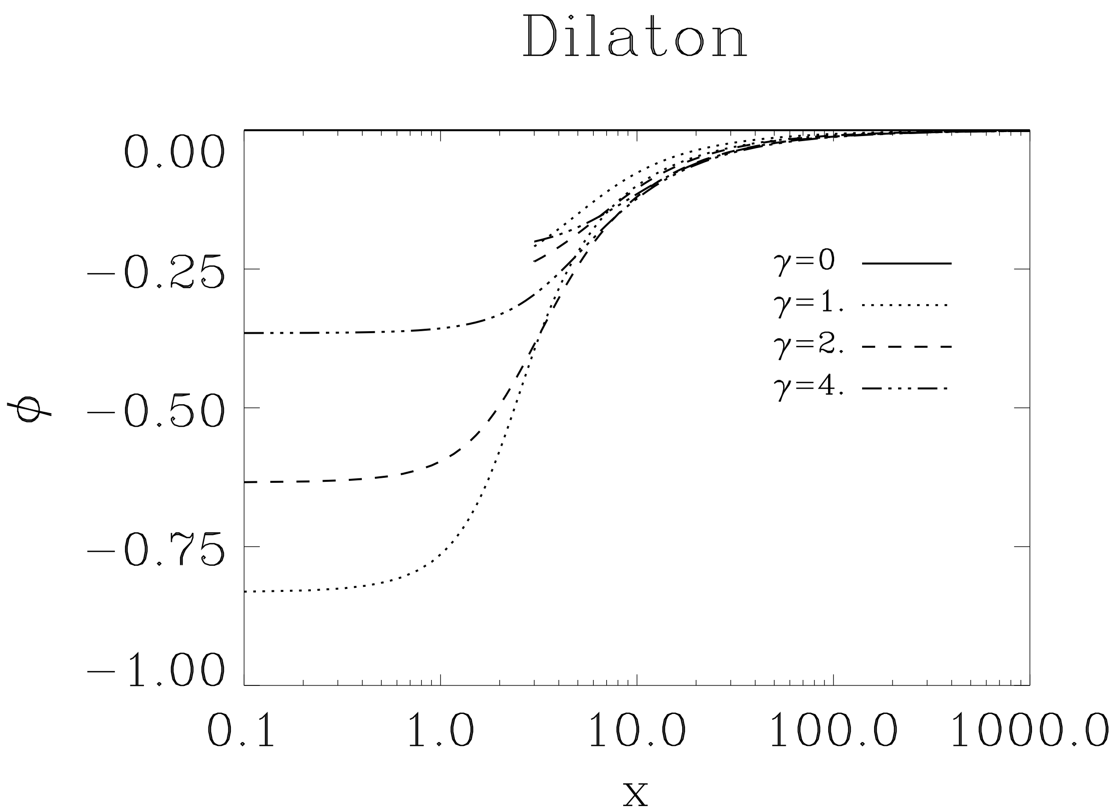

Figure 3: The function $\phi(x)$ is shown for the regular solution and for the black hole solution with horizon $x_{\rm H} = 3$ for the dilaton coupling constants $\gamma = 0$ (solid), $\gamma = 1$ (dotted), $\gamma = 2$ (dashed) and $\gamma = 4$ (tripledot-dashed) as a function of x.

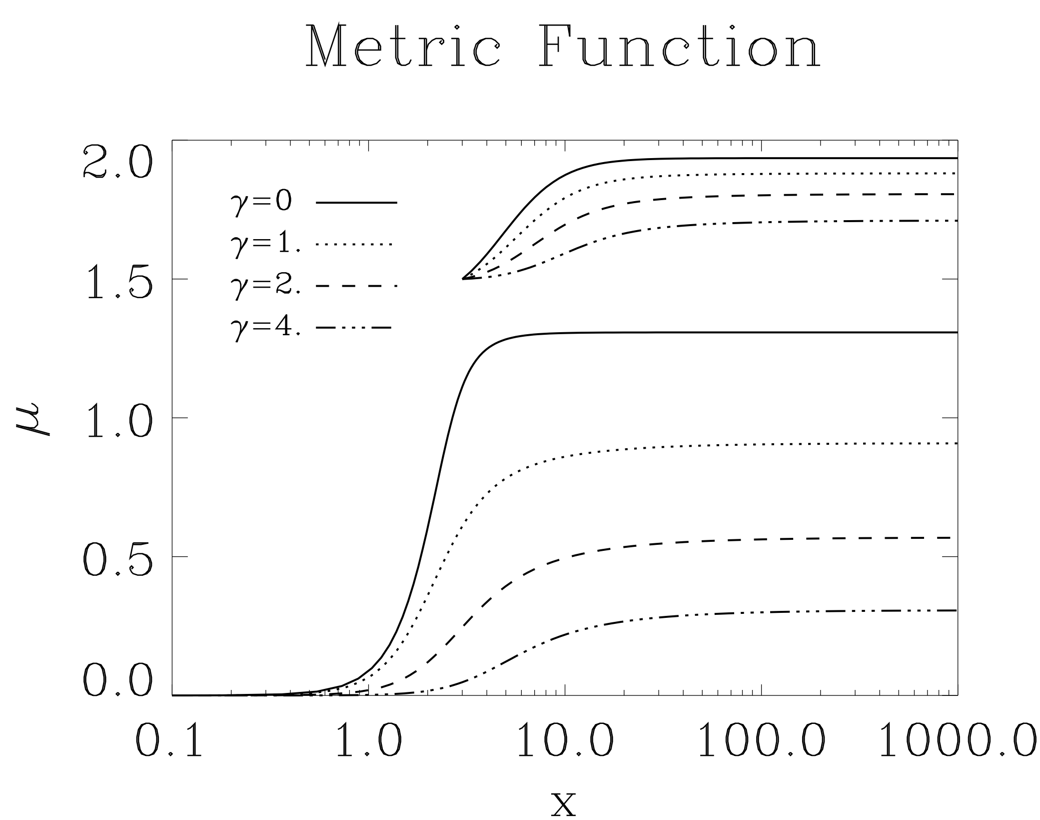

Figure 4: The function $\mu(x)$ is shown for the regular solution and for the black hole solution with horizon $x_{\rm H} = 3$ for the dilaton coupling constants $\gamma = 0$ (solid), $\gamma = 1$ (dotted), $\gamma = 2$ (dashed) and $\gamma = 4$ (tripledot-dashed), as a function of x.

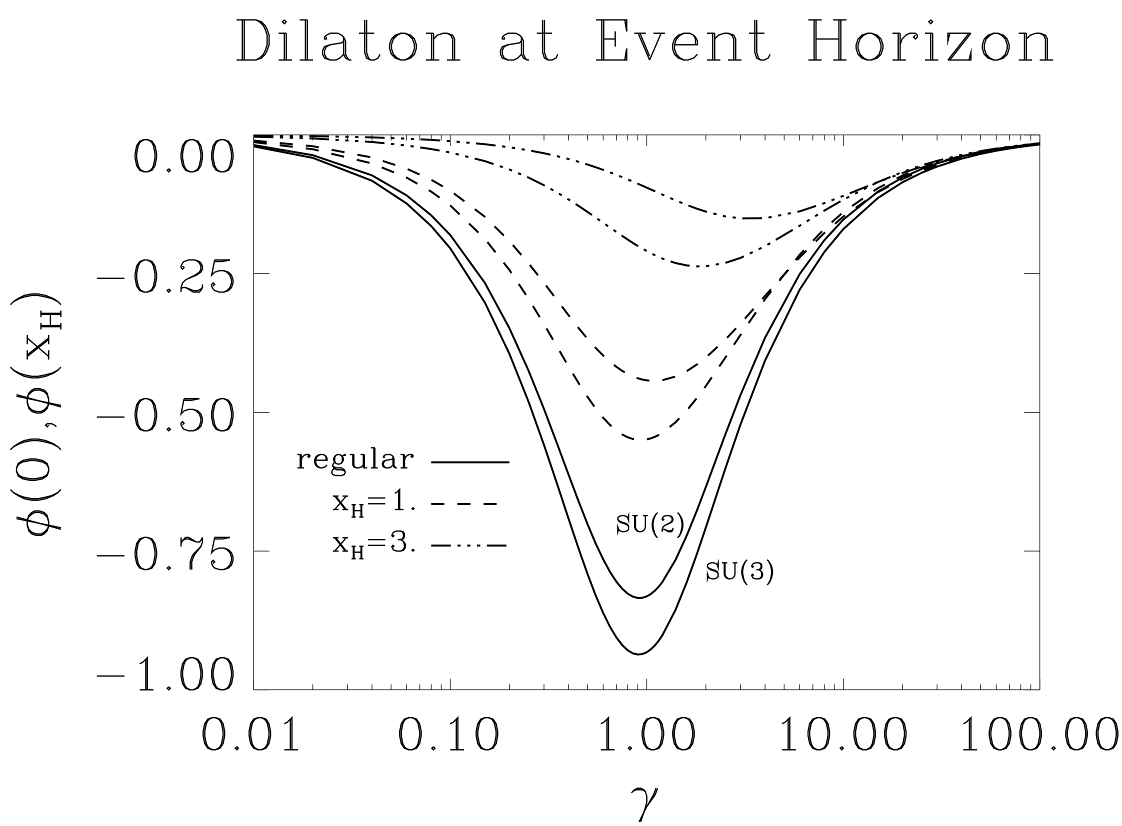

Figure 5: The dilaton field at the origin, $\phi(0)$, for the regular solution (solid) and the dilaton field at the horizon, $\phi(x_{\rm H})$, for the black hole solutions with horizons $x_{\rm H} = 1$ (dashed) and $x_{\rm H} = 3$ (tripledot-dashed) are shown as a function of the dilaton coupling constant γ. The lower curves correspond to SO(3) solutions, while the upper curves correspond to SU(2) solutions.

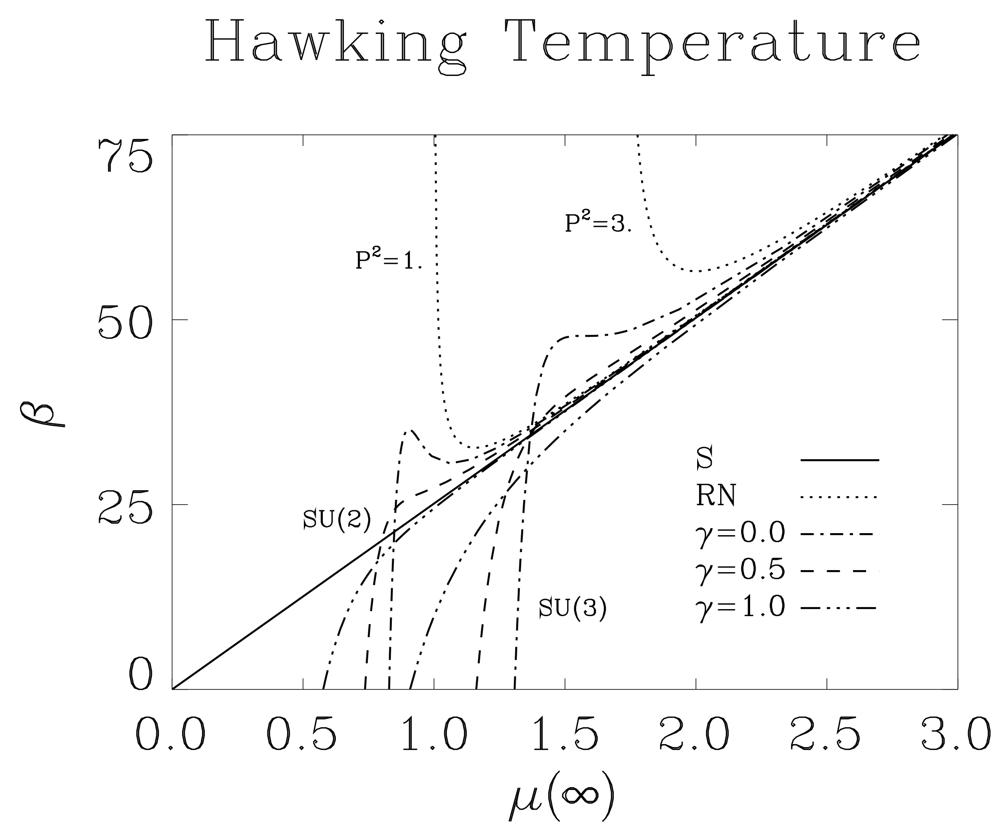

Figure 6: The inverse of the dimensionless Hawking temperature $\beta = T^{-1}$ is shown as a function of the dimensionless black hole mass $\mu(\infty)$ for the lowest SU(2) and SO(3) solutions and for dilaton coupling constants $\gamma = 0$ (dot-dashed), $\gamma = 0.5$ (dashed), and $\gamma = 1$ (tripledot-dashed). Also shown is the inverse temperature for the Schwarzschild (solid) and Reissner-Nordstrøm black holes (dotted) with magnetic charge P.